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International Journal of Solids and Structures 41 (2004) 7063–7074
INTERNATIONAL JOURNAL OF SOLIDS and STRUCTURES
www.elsevier.com/locate/ijsolstr

Simplified model for prediction of dynamic damage and fracture of ductile materials

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Received 26 October 2003; received in revised form 25 May 2004
Available online 20 July 2004

Abstract

A simplified model for prediction of dynamic damage and fracture of ductile material has been proposed. The plastic flow of matrix and the void growth are dealt with separately after we show that two separated loading sub-surfaces and corresponding normality rules for matrix and damage exist. The equation of the loading sub-surface for the matrix plastic flow is derived by the means of introducing a mapping damage-free solid of matrix material, whose constitutive relation is supposed to have been determined via Hopkinson bar experiments. Based on the results of recovered experiments the law of damage evolution is phenomenally established. The model has been applied to predict some spall experiments carried out on tantalum, and results show it predicts the experiments very well.

Keywords: Dynamic damage; Fracture; Failure; Plasticity

1. Introduction

Dynamic damage and fracture occurs in a wide range of technologically important applications, for examples: high-speed machining, crash-worthiness of vehicles, armor penetration, striking of dust particles on aerospace vehicles and satellites. It has long been recognized that in the cases of dynamic loading conditions the material behaves are different with those under static loading conditions. Under static loading, the failure or fracture of materials was demonstrated to be the consequence of fatigue and the propagation of pre-existed macro-scale cracks (Lawn, 1993). In contrast, dynamic failure and fracture of materials was observed to result from the accumulation of damage that is characterized by the nucleation, growth and coalescence of microcracks and microvoids (Seaman et al., 1976; Curran et al., 1987). So

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0020-7683/$ - see front matter
doi:10.1016/j.ijsolstr.2004.05.061

modeling of the response of materials under dynamic loading is, in fact, a multi-scale problem. In order to link micro-damage, macro-deformation and the final failure of materials, Kachanov (1958) put forward the concept of internal variable of damage, which was defined as the relative area of voids in a cross section. The significance of this concept is that it enables us to study the mechanic properties of damaged materials with continuum theories. In 1970s the concept was widely accepted and extended to describe the dynamic behaves of materials under high rate loadings such as explosions and high-velocity impactions. Afterwards, numerous models have been proposed, for example, Davision and Stevens (1972, 1973), Seaman et al. (1976), Curran et al. (1987), Johnson (1981), Steinberg et al. (1980, 1989), Johnson and Addessio (1989), Perzyna (1986a,b), Rajendran et al. (1989, 1991), Krajcinovic (1989, 2000), Bai et al. (1991, 1992, 1997, 2000), Wang (1997), Hansen and Schreyer (1994), Cauvin and Testa (1999), Li (2000), Clifton (2000), Ireman et al. (2003) and Rashid et al. (2003). For review papers, one can refer to Voyiadjis et al. (2002), Meyers (1994) and Curran et al. (1987) as examples. Although plenty of models have been proposed, it has been found that the prediction of material response to dynamic loading is also insufficient, further works are required.

In this paper, we will present a novel practical model for prediction of dynamic damage and fracture of ductile materials. But prior to all, we will first show that there exist two separated loading sub-surfaces and corresponding normality rules for the matrix plastic flow and the damage evolution. The advantage of the separation is that it makes the frame of the model applicable for both ductile and brittle materials. In addition it allows us to change the kinetic equations for the damage evolution and the matrix plastic flow without a large amount of modification of the codes. Next in our paper, the frame of our model will be presented. After that, the rate-dependent loading sub-surface for the matrix flow will be introduced and the equation of evolution of damage will be developed. At last some examples of simulation of plate impacting experiments will be exhibited and a summary will be given.

2. Existence of separated sub-surfaces for the matrix plastic flow and the damage evolution

In elastic–plastic mechanics, there is a basic rule, i.e., the normality rule, which is resulted from the Drucker's postulate (1952). In fact, for a dissipative mechanic system Li (1999) on the view of thermodynamics demonstrated that there exists a general normality rule between any thermodynamic strain and thermodynamic stress. So in the damage and plastic deformation coupled dissipative system, the following equation holds

$$\mathrm{d}\varepsilon_{ij}^{ir} = \mathrm{d}\lambda \frac{\partial \phi}{\partial \sigma_{ij}}, \tag{1}$$

where $\mathrm{d}\varepsilon_{ij}^{ir}$ $(i, j = 1, 2, 3)$ are the increments of the components of total irreversible strain; $\mathrm{d}\lambda$ is a positive parameter; σ_{ij} $(i, j = 1, 2, 3)$ are the components of stress tensor $(\vec{\vec{\sigma}})$; $\phi = 0$ is the equation for the loading surface (dissipative surface in the Li's paper (Li, 1999)). By multiplicative decomposition, the total irreversible strain can be divided into two parts, that is

$$\varepsilon_{ij}^{ir} = \varepsilon_{ij}^{\mathrm{m}} + \varepsilon_{ij}^{\mathrm{D}}. \tag{2}$$

$\varepsilon_{ij}^{\mathrm{m}}$ are the contributions of the matrix to total irreversible strain due to plastic flow. $\varepsilon_{ij}^{\mathrm{D}}$ are the contributions of damage due to the emergence of voids and cracks. So Eq. (1) can be rewritten as

$$\mathrm{d}\varepsilon_{ij}^{\mathrm{m}} + \mathrm{d}\varepsilon_{ij}^{\mathrm{D}} = \mathrm{d}\lambda \frac{\partial \phi}{\partial \sigma_{ij}}. \tag{3}$$

Because the left-hand side of Eq. (3) is consisted of two separated parts, we can also divide the right-hand side into two separated parts. Then the following equations can be derived

$$\phi = \phi_{\mathrm{m}} + \phi_{\mathrm{D}} = 0, \tag{4}$$

$$\phi_{\mathrm{m}} = 0, \tag{5}$$

$$\phi_{\mathrm{D}} = 0, \tag{6}$$

$$\mathrm{d}\varepsilon_{ij}^{\mathrm{m}} = \mathrm{d}\lambda \frac{\partial \phi_{\mathrm{m}}}{\partial \sigma_{ij}}, \tag{7}$$

$$\mathrm{d}\varepsilon_{ij}^{\mathrm{D}} = \mathrm{d}\lambda \frac{\partial \phi_{\mathrm{D}}}{\partial \sigma_{ij}}. \tag{8}$$

The implications of Eqs. (5)–(8) are very clear, which can be concluded as follows. The plastic flow of matrix and the evolution of damage have their own loading sub-surfaces and both are subjected to the normality rules of themselves. In above equations, Eqs. (5) and (6) are the sub-surfaces for matrix and damage, respectively. In previous works, several authors, for example Lubarda et al. (1994), Hansen and Schreyer (1994), Li (1999) have assumed the decomposition of loading surface. From the above analysis, we can see, in fact it is directly resulted from the decomposition of the strain, so it is rational.

3. Frame of model

First, we should emphasize that in this paper we limit ourselves to study void type damage, which can be described by a scalar variable, and usually defined as the fraction of void volume with respect to total (Rajendran et al., 1989; Feng et al., 1997), i.e., $D = V_{\mathrm{v}}/V$. D is the damage variable, V_{v} is the volume of voids, V is the total volume of the composite (matrix and voids). It is well accepted that the growth of void is motivated by the tensile volumetric stress. So we generally assume the evolution of damage can be expressed as

$$\dot{D} = \dot{D}(P, D, X), \tag{9}$$

where $\dot{D}$ is the rate of D. P is the pressure (defined as negative volumetric stress). X denotes the other related internal state variables.

On the other hand, if we assume that the plastic deformation of matrix would not cause the change of volume, that is

$$\varepsilon_{ii}^{\mathrm{m}} \equiv 0. \tag{10}$$

Under this assumption, the plastic deformation of matrix is motivated by the energy of shear deformation. So from the view of energy we may generally write the function of the loading sub-surface for the matrix plastic flow as

$$\phi_{\mathrm{m}} = \frac{1}{1-D} \times \frac{J_2}{2G_D} - \psi_{\mathrm{m}} = 0, \tag{11}$$

where $\frac{1}{1-D} \times \frac{J_2}{2G_D}$ is the energy density of shear deformation in the material matrix; J_2 is the second invariant of the stress tensor; G_D is the shear modulus of the composite, which is degraded by damage according to the following equation (Mackenzie, 1950)

$$G_D = G_0(1-D)\left(1 - D\frac{6B_0 + 12G_0}{9B_0 + 8G_0}\right), \tag{12}$$

where B_0, G_0 are the bulk and shear modulus of the undamaged matrix material, respectively. Because J_2 is independent on the selection of coordinate system, ψ_m should be a function in terms of invariants of internal variables, thus it may takes the following general form

$$\psi_{\mathrm{m}} = \psi_{\mathrm{m}}(\varepsilon_{\mathrm{m}}, \dot{\varepsilon}_{\mathrm{m}}, D, T, X), \tag{13}$$

where ε_{m} denotes the invariants of matrix plastic strain, $\dot{\varepsilon}_{\mathrm{m}}$, the rate of ε_{m}, T is temperature and X denotes other related internal state variables.

Modeling the material response to a define loading is to solve the problem i.e., by knowing velocity field, stress field, strain field, damage distribution and other internal variables to calculate their corresponding value at next time step. With the Eqs. (9) and (11), this can be easily realized. See the following:

Total strain rate ($\dot{\varepsilon}_{ij}$):

$$\dot{\varepsilon}_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right). \tag{14}$$

Evolution of velocity ($\dot{u}_i$):

$$\dot{u}_i = \frac{1}{\rho}\frac{\partial \sigma_{ij}}{\partial x_j}, \tag{15}$$

ρ is the mass density of the composite.

Evolution of damage:

$$\dot{D} = \dot{D}(P, D, X). \tag{9}$$

Plastic strain rate of matrix ($\dot{\varepsilon}_{ij}^{m}$):

$$\dot{\varepsilon}_{ij}^{m} = \dot{\lambda}\frac{\partial \phi_{\mathrm{m}}}{\partial \sigma_{ij}}, \tag{16}$$

where $\dot{\lambda}$ is determined by Eqs. (11)–(13) and (16).

Irreversible strain rate contributed by damage ($\dot{\varepsilon}_{ij}^{D}$):

Because

$$\dot{D} = \frac{\dot{V}_{\mathrm{v}}}{V} - \frac{V_{\mathrm{v}}}{V^2}\dot{V} = \dot{\varepsilon}_{ii}^{D} - D\dot{\varepsilon}_{ii}.$$

so:

$$\begin{aligned} \dot{\varepsilon}_{ii}^{D} &= \dot{D} + D\dot{\varepsilon}_{ii} \\ \dot{\varepsilon}_{11}^{D} &= \dot{\varepsilon}_{22}^{D} = \dot{\varepsilon}_{33}^{D} = \frac{1}{3}(\dot{D} + D\dot{\varepsilon}_{ii}), \end{aligned} \tag{17}$$

$$\dot{\varepsilon}_{ij}^{D} = 0 \quad (i \neq j). \tag{18}$$

Elastic strain rate ($\dot{\varepsilon}_{ij}^{e}$):

$$\dot{\varepsilon}_{ij}^{\mathrm{e}} = \dot{\varepsilon}_{ij} - \dot{\varepsilon}_{ij}^{\mathrm{m}} - \dot{\varepsilon}_{ij}^{\mathrm{D}}. \tag{19}$$

Deviatoric stress (S_{ij}):

$$S_{ij} = 2G_D\tau_{ij}^{\mathrm{e}}, \tag{20}$$

where τ_{ij}^{e} is the deviatoric elastic strain.

The increasing rate of specific internal energy per volume ($\dot{E}$):

$$\dot{E} = S_{ij}\dot{\varepsilon}_{ij}^{\mathrm{m}} + P\dot{\varepsilon}_{ii}^{\mathrm{D}}. \tag{21}$$

Temperature increasing rate ($\dot{T}$):

$$\dot{T} = \frac{\beta}{\rho C_V}\dot{E}, \tag{22}$$

where C_V is the specific heat of matrix; β is the so-called conversion coefficient, whose value was determined to be about 0.9 empirically.

As to the pressure P, it is determined by the equation of state. We may generally express it as

$$P = P(\rho, E, D). \tag{23}$$

From the above, we can see if we need to change the detail expressions of Eqs. (9), (13) and (23), we only need to rewrite a few external functions in the codes. This is valuable in applications.

4. Sub-surface of matrix

The general expression for the loading sub-surface for the matrix plastic flow has been shown in above as equation (11), in which J_2 is the second invariant of the stress tensor, which is defined as

$$J_2 = \frac{1}{2}S_{ij}S_{ij} = \frac{1}{6}[(\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2]. \tag{24}$$

For the purpose to find out the express for ψ_{m}, we introduce a mapping damage-free solid of matrix material, whose plastic strain, mass density and specific internal energy are defined to be always identical to those of the matrix of the composite. We suppose the stress of the mapping solid ($\sigma_{ij}^{\mathrm{map}}$) follow the assumption made by Carroll and Holt (1972)

$$P = P_{\mathrm{map}}\frac{\rho}{\rho_{\mathrm{m}}}, \tag{25}$$

where P is the macroscopic pressure in the composite, P_{map} is the negative volumetric component of $\sigma_{ij}^{\mathrm{map}}$, ρ and ρ_{m} are the density of the composite and matrix, respectively. Thus we may obtain symmetrically

$$\sigma_{ij}^{\mathrm{map}} = \frac{\sigma_{ij}}{1 - D}, \tag{26}$$

$$\frac{1}{1 - D} \times \frac{J_2}{2G_D} = (1 - D)\frac{J_2^{\mathrm{map}}}{2G_D}. \tag{27}$$

J_2^{map} is the second invariant of the stress $\sigma_{ij}^{\mathrm{map}}$. Providing the constitutive relation for the damage-free matrix material has been determined via the experiments of Hopkinson bar (Davies and Hunter, 1963; Kolsky, 1949)

$$Y = Y(\varepsilon_{\mathrm{p}}, \dot{\varepsilon}_{\mathrm{p}}, X). \tag{28}$$

ε_{p}, $\dot{\varepsilon}_{\mathrm{p}}$, X are plastic strain, strain-rate and other concerning internal variables, respectively, Then we can write, as usual, the loading surface of Von Mises type for the mapping solid as (Dowell, 1992).

$$\frac{3}{2}\sum_{i,j}[S_{ij}^{\mathrm{map}} - h_{ij}(\varepsilon_{ij}^{\mathrm{m}})][S_{ij}^{\mathrm{map}} - h_{ij}(\varepsilon_{ij}^{\mathrm{m}})] = Y^2(\bar{\varepsilon}_{\mathrm{m}}, \dot{\bar{\varepsilon}}_{\mathrm{m}}, X). \tag{29}$$

$S_{ij}^{\rm map}$ are the components of the deviatoric stress of the mapping solid, $h_{ij}(\varepsilon_{ij}^{\rm m})$ represents the plastic strain hardening functions. When $h_{ij} \equiv 0$, it is isotropic type, otherwise it is kinematic type. $\bar{\varepsilon}_{\rm m}$, $\dot{\bar{\varepsilon}}_{\rm m}$ are the effective strain and strain rate, respectively. For simplicity we let $h_{ij}(\varepsilon_{ij}^{\rm m}) = h(\bar{\varepsilon}_{\rm m})$ approximately, then Eq. (29) can be rewritten as

$$J_2^{\rm map} = \frac{1}{3} Y^2(\bar{\varepsilon}_{\rm m}, \dot{\bar{\varepsilon}}_{\rm m}, X) - \frac{3}{2} h^2(\bar{\varepsilon}_{\rm m}). \tag{30}$$

Substituting Eq. (30) into Eq. (27), we get the general function of the loading sub-surface for the matrix plastic flow

$$J_2 = (1 - D)^2 \left[\frac{1}{3} Y^2(\bar{\varepsilon}_{\rm m}, \dot{\bar{\varepsilon}}_{\rm m}, X) - \frac{3}{2} h^2(\bar{\varepsilon}_{\rm m}) \right]. \tag{31}$$

For example, if we choose Johnson–Cook model (Johnson and Cook, 1985) for the constitutive relation, then

$$Y = [A + B(\bar{\varepsilon}_{\rm m})^n] \left[1 + C \ln \left(\frac{\dot{\bar{\varepsilon}}_{\rm m}}{\dot{\varepsilon}_0} \right) \right] \left[1 - \left(\frac{T - T_{\rm r}}{T_{\rm m} - T_{\rm r}} \right)^M \right], \tag{32}$$

where, A, B, C, n, M are material parameters. $\dot{\varepsilon}_0$ is the normalized parameter. $T_{\rm r}$ is the room temperature. $T_{\rm m}$ is the melt temperature of the material.

5. Damage evolution

It is well known that the evolution of voids and cracks is generally divided into three aspects: nucleation, growth and coalescence. A great number of references have been devoted to this problem, for instance, Carroll and Holt (1972), Johnson (1981). Curran et al. (1987), Stumpf and Hackl (2003) and Wua et al. (2003).

Bai et al. (1992, 1997, 2000), in the light of statistical mesoscopic damage mechanics, established a fundamental equation for micro-damage evolution.

$$\frac{\partial n}{\partial t} + \sum_{i=1}^{I} \frac{\partial (n \cdot Q_i)}{\partial q_i} = n_{\rm N} - n_{\rm A}, \tag{33}$$

where n is the number density of micro-damage in phase space, t denotes time, q_i are the independent variables describing the state of micro-damages, Q_i are the rate of variable q_i, $n_{\rm N}$ and $n_{\rm A}$ are the nucleation and annihilation rate densities of micro–damage, respectively. For voids, Eq. (33) can be simplified.

$$\frac{\partial n}{\partial t} + \frac{\partial (n \cdot \dot{v})}{\partial v} = n_{\rm N} - n_{\rm A}, \tag{34}$$

v is the volume of void.

Nowadays modeling the coalescence of voids remains a giant problem. So an effective way for dealing with the problem is to assume that before certain critical damage $D_{\rm c}$ is reached, no coalescence occurs. After that voids are rapidly coalesced to separate the sample. Therefore, before $D_{\rm c}$ is reached, the following equation holds

$$\dot{D} = \frac{\dot{V}_{\rm v}}{V} - \frac{V_{\rm v}}{V^2} \dot{V} = \frac{(\dot{V}_{\rm v})_{\rm N}}{V} + \frac{(\dot{V}_{\rm v})_{\rm G}}{V} - D \dot{\varepsilon}_b. \tag{35}$$

$(\dot{V}_{\rm v})_{\rm N}$ is the contribution of the nucleation to the rate of increment of the total volume of voids, $(\dot{V}_{\rm v})_{\rm G}$ is that of the growth of voids, $\dot{\varepsilon}_b$ is the rate of the volume strain of the composite. Referring to the work of Curran et al. (1987), we write the nucleation rate of voids in per matrix mass as

$$\dot{n}_{\rm N} = \begin{cases} N_0 \exp\left(\dfrac{P_{\rm m} - P_{\rm n0}}{P_{\rm n}}\right) & P_{\rm m} \geqslant P_{\rm n0} \\ 0 & P_{\rm m} < P_{\rm n0}, \end{cases}. \tag{36}$$

where $P_{\rm n0}$ is the threshold stress for void nucleation, $P_{\rm n}$ is a characteristic parameter with stress unit. N_0, $P_{\rm n0}$, $P_{\rm n}$ are material constants. If we assume the average size of nucleated voids is also material constant, whose radius is $R_{\rm N}$, then we get:

$$\frac{(\dot{V}_{\rm v})_{\rm N}}{V} = \begin{cases} \dfrac{4\pi}{3} R_{\rm N}^3 \rho N_0 \exp\left(\dfrac{P_{\rm m} - P_{\rm n0}}{P_{\rm n}}\right) & P_{\rm m} \geqslant P_{\rm n0} \\ 0 & P_{\rm m} < P_{\rm n0} \end{cases}. \tag{37}$$

Curran et al. (1987) and Bai et al. (2000) had found, via recovered experiments, that in the process of growth the void number distribution always fulfills a certain function, which is generally in terms of R/R_0. This must lead to the conclusion [Curran et al., 1987]

$$\frac{\dot{R}}{R} = \frac{\dot{R}_0}{R_0} = C, \tag{38}$$

where, R is the radius of void, R_0 is a time-dependent parameter that characterizes the distribution, C is a constant that is independent of R. Eq. (38) implies for every void

$$\dot{v} = 4\pi R^2 \dot{R} = 3Cv. \tag{39}$$

On the other hand, we have

$$\frac{(\dot{V}_{\rm v})_G}{V} = \frac{1}{V}\frac{\rm d}{{\rm d}t}\int v n(v,t)_G {\rm d}v = \frac{1}{V}\left\{\int\left[\dot{v} n(v,t)_G + v\frac{\partial n(v,t)_G}{\partial t} + v\frac{\partial n(v,t)_G}{\partial v}\dot{v}\right]{\rm d}v + \int v n(v,t)_G {\rm d}\dot{v}\right\}. \tag{40}$$

$n(v,t)_G$ is the number density of voids after the nucleation is excluded. If we limit ourselves to study the voids growth, Eq. (34) becomes

$$\frac{\partial n(v,t)_G}{\partial t} + \frac{\partial (n \cdot \dot{v})}{\partial v} = 0. \tag{41}$$

Solving Eqs. (39)–(41), we obtain

$$\frac{(\dot{V}_{\rm v})_G}{V} = 3CD. \tag{42}$$

Feng et al. (1997) from the point of energy balance, had demonstrated

$$C = \frac{C_{\rm b}^{\rm m}}{4B_{\rm m}\lambda}(P_{\rm m}^2 - P_{\rm g}^2). \tag{43a}$$

$P_{\rm g}$ is the threshold stress for void growth, which is material constant, $C_{\rm b}^{\rm m}$ and $B_{\rm m}$ are the bulk sound velocity and bulk modulus of matrix, respectively. λ is the specific fracture energy. In their deduction, they had not considered the fact that the medium around a void is also damaged. When taking this point into consideration, Eq. (43a) should be modified to

$$C = \frac{C_{\rm b}}{4B_{\rm m}\lambda}(P_{\rm m}^2 - P_{\rm g}^2)(1 - D), \tag{43b}$$

C_b is the bulk sound velocity of the composite. Furthermore, we consider B_m is degraded by damage according to Mackenzie's relation (Mackenzie, 1950)

$$B_m = \frac{4G_0B_0(1-D)}{4G_0+3B_0D}, \tag{44}$$

B_0, G_0 are the bulk and shear modulus of the undamaged matrix material. So we finally get the damage evolution equation.

$$\dot{D} = \begin{Bmatrix} \frac{4\pi}{3}R_N^3\rho N_0 \exp\left(\frac{P_m - P_{n0}}{P_n}\right) & P_m \geqslant P_{n0} \\ 0 & P_m < P_{n0} \end{Bmatrix} + \begin{Bmatrix} \frac{3C_b}{4B_m\lambda}(P_m^2 - P_g^2)D(1-D) & P_m \geqslant P_g \\ 0 & P_m < P_g \end{Bmatrix} - D\dot{\varepsilon}_b. \tag{45}$$

6. Equation of state

We simply divide the matrix pressure (P_m) into the cold part and the thermal part, for which we select Murnaghan equation and Gruneisen equation, respectively, then

$$P_m = \frac{B_0}{B_0'}\left[\left(\frac{\rho_m}{\rho_0}\right)^{B_0'} - 1\right] + \frac{\gamma E}{(1-D)}, \tag{46}$$

where, B_0' is the derivative of B_0 with respect to pressure, ρ_m and ρ_0 are the mass density of the matrix at current and zero-pressure states. γ is the Gruneisen ratio of the matrix material. E is the specific thermal energy of the composite per volume. Then the pressure of the composite is calculated by Eq. (25).

7. Destabilizing factor

In order to describe the rapid destabilizing behaves of the damaged materials, which emerges when the microvoids or microcracks are very close to be totally coalesced to separate the solid, we, similar to Feng et al. (1997), would like to define a destabilizing factor

$$F(D) = \exp\left[-\left(\frac{D}{D_1}\right)^{\alpha}\right], \tag{47}$$

D_1 and α are two empirical constants that should be determined by trial calculations. We suppose the moduli B_m, G_D, the plastic strain hardening function $h(\bar{\varepsilon}_m)$ and the yield strength Y will be decreased quickly by $F(D)$ after $D > D_1$, so Eqs. (12), (31) and (44) should be revised as follows:

$$G_D = G_0(1-D)\left(1 - D\frac{6B_0+12G_0}{9B_0+8G_0}\right)F(D), \tag{12'}$$

$$J_2 = (1-D)^2\left[\frac{1}{3}Y^2(\bar{\varepsilon}_m, \dot{\bar{\varepsilon}}_m, X) - \frac{3}{2}h^2(\bar{\varepsilon}_m)\right]F^2(D), \tag{31'}$$

$$B_m = \frac{4G_0B_0(1-D)}{4G_0+3B_0D}F(D). \tag{44'}$$

8. Simulation examples

We had carried out three shots of impact experiments on pure tantalum targets with a gas gun, the free surface velocity profiles had been measured with VISAR (Barker and HollenBack, 1972). The experiments sets are shown in Fig. 1. What we want to point out is that the projectile is consisted of two layers. The left one (aluminium alloy) acts as the supporter for the right one (tantalum). The parameters for the three shot experiments are given in Table 1.

The constitutive relations of the aluminium alloy and the tantalum had been measured with Hopkinson bar technique. The relation of the aluminium alloy was fitted to J–C model (refer to Eq. (32)), and that of the tantalum was fitted to a modified J–C model shown in Eq. (48).

$$Y = [A + B(\bar{\varepsilon}_{\mathrm{m}})^n]\left(\frac{\dot{\bar{\varepsilon}}_{\mathrm{m}}}{\dot{\varepsilon}_0}\right)^C\left[1 - \left(\frac{T - T_{\mathrm{r}}}{T_{\mathrm{m}} - T_{\mathrm{r}}}\right)^M\right]. \tag{48}$$

The corresponding parameters for the constitutive relations are shown in Table 2.

In simulations, the parameters for the equation of state were determined by virtue of Hugoniot data (Jing, 1999). And other damage-related parameters were determined by trial calculations. The trial calculations were carried out on the experiment shot 3. Then the other two experiments of shot 2 and shot 1

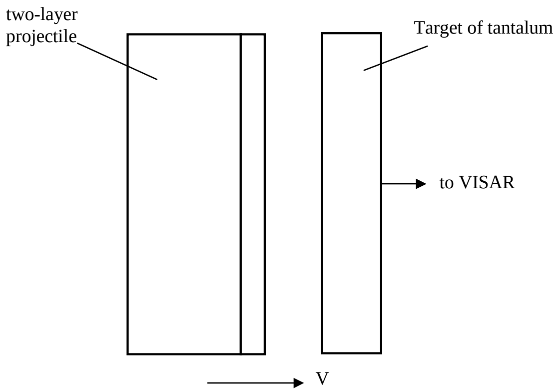

Fig. 1. The experimental sets.

Table 1
Parameters for the experiments

No.	Target (Ta)	Left projectile (aluminium alloy)	Right projectile (Ta)	Impact velocity (m/s)
	Width (mm)	Width (mm)	Width (mm)	
Shot 1	3.994	8	1.4511	294
Shot 2	4.066	8	1.631	522
Shot 3	4.057	8	1.452	705

Table 2
Parameters for the constitutive relations of the materials

Material	A (GPa)	B (GPa)	$\dot{\varepsilon}_0$ (s^{-1})	T_r (K)	T_m (K)	C	n	M
Aluminium alloy	0.31	1.13	1	300	960	0.015	0.69	0.88
Tantalum	0.34	0.26	4×10^{-4}	300	3269	0.042	0.32	0.88

were predicted theoretically. Because the left projectile (aluminium alloy of 8 mm width) is just a supporter for the right one, so we did not take into consideration of the damage of the left projectile in the simulations. Table 3 shows the other parameters used in the three simulations. The comparisons of the free

Table 3
The parameters used in the simulations

	ρ_0 (kg/m^3)	B_0 (Pa)	B_0'	G_0 (Pa)	γ	C_v (J kg^{-1} K^{-1})			
Tantalum	16.67×10^3	1.96×10^{11}	4.2	6.9×10^{10}	1.58	0.14×10^3			
Aluminium alloy	2.7×10^3	7.55×10^{11}	4.32	2.65×10^{10}	1.78	0.9×10^3			
$h(\bar{\varepsilon}_m)$ (Pa)	N_0 (kg^{-1} s^{-1})	P_{n0} (Pa)	P_n (Pa)	R_N (m)	P_g (Pa)	λ (J/m^2)	D_1	α	D_c
0	1.0×10^{10}	6.0×10^8	2.0×10^8	1.0×10^{-7}	6×10^8	8.5×10^3	0.06	4	0.2
0									

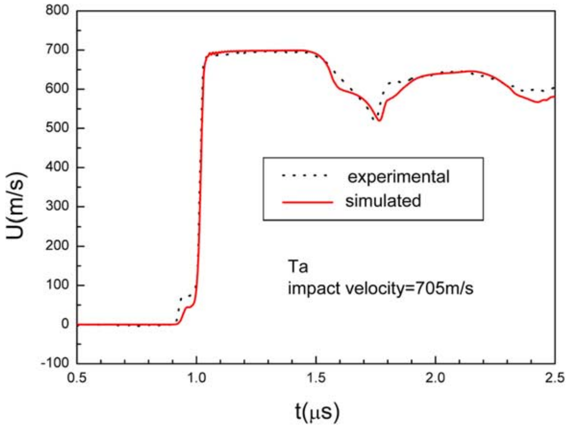

Fig. 2. The comparison of the velocity profile of the free surface between the theoretical simulation and the experimental measurement for shot 3.

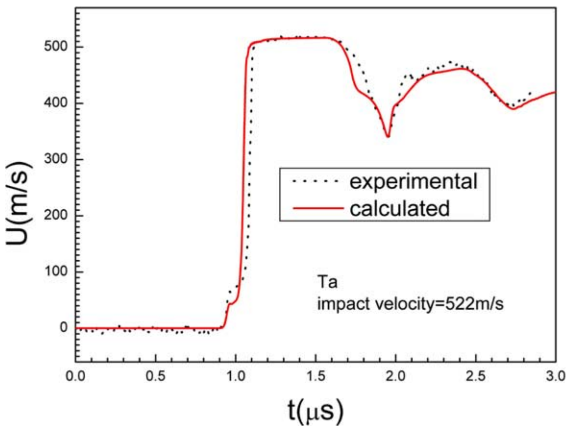

Fig. 3. The comparison of the velocity profile of the free surface between the theoretical prediction and the experimental measurement for shot 2.

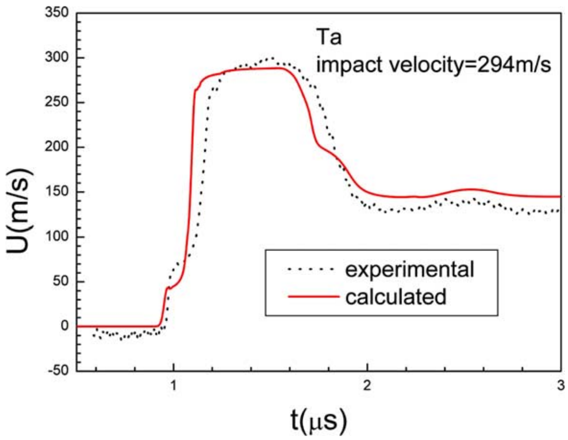

Fig. 4. The comparison of the velocity profile of the free surface between the theoretical prediction and the experimental measurement for shot 1.

surface velocity profiles between that of calculated and that of experiments are shown from Figs. 2–4, which show that the above model predicts experiments very well.

9. Summary

A simplified model for prediction of the dynamic damage and fracture of ductile materials has been proposed. In doing so, we firstly succeeded to show that two separated loading sub-surfaces and the corresponding normality rules for the matrix plastic flow and the damage evolution exist. The advantage of the separation is that it enables us to derive a universal frame for modeling the dynamic damage and fracture behaves of materials. After that, we introduced a mapping damage-free solid of matrix material, and by assuming that the stress of the mapping solid follows the Carroll and Holt's relation (Carroll and Holt, 1972), we derived the function of the loading sub-surface for the matrix plastic flow. Next we deduced the equation of the damage evolution, and introduced a destabilizing factor to describe the rapid destabilizing behaves of materials, which occurs before the microvoids are finally coalesced. At last three shots of experiments were simulated, and results show the model predicts the experiments very well.

Acknowledgements

We would like to thank the support of the Science Foundation of China Academy of Engineering Physics under Grant no 2000Z0307.

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